HOMEWORK 7 – ANSWERS TO (MOST) PROBLEMS

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Section 3.10: Linear approximations and differentials

3.10.2. $L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

3.10.11.

(a) $dy = (2x\sin(2x) + 2x^2\cos(2x))dx$

(b) $dy = \frac{1}{\sqrt{1+t^2}}\left(\frac{t}{\sqrt{1+t^2}}\right)dt = \frac{t}{1+t^2}dt$

3.10.15.

(a) $dy = \frac{1}{10}e^{\frac{x}{10}}dx$
(b) $dy = \frac{1}{10}(0.1) = 0.01$

3.10.21. $\Delta(y) = y(5) - y(4) = \frac{2}{5} - \frac{2}{4} = -\frac{1}{10} = -0.1$
$dy = -\frac{2}{4^2}(1) = -\frac{1}{8} = -0.125$

3.10.35. $l = 2\pi r = 84$, so $r = \frac{84}{2\pi} = \frac{42}{\pi}$. We know $dl = 0.5$, so $2\pi dr = 0.5$, so $dr = \frac{0.5}{2\pi} = \frac{1}{4\pi}$

(a) $S = 4\pi r^2$, so $dS = 8\pi r dr = 8\pi\frac{42}{\pi}\frac{1}{4\pi} = \frac{84}{\pi}$. Also the relative error is $\frac{dS}{S} = \frac{8\pi r dr}{4\pi r^2} = \frac{2dr}{r} = \frac{1}{2\pi} \times \frac{\pi}{42} = \frac{1}{84} \approx 0.012$
(b) $V = \frac{4}{3}\pi r^3$, so $dV = 4\pi r^2 dr = 4\pi\frac{42^2}{\pi^2} \times \frac{1}{4\pi} = \frac{1764}{\pi^2} \approx 179$. Also the relative error is $\frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = \frac{3dr}{r} = \frac{\frac{3}{4\pi}}{\frac{42}{\pi}} = \frac{3}{168} = \frac{1}{56} \approx 0.018$

3.10.40. $dF = 4kR^3dR$, so:

$$\frac{dF}{F} = \frac{4kR^3dR}{F} = \frac{4kR^3dR}{F} = \frac{4kR^3dR}{kR^4} = 4\frac{dR}{R}$$

And when $\frac{dR}{R} = 0.05$, $\frac{dF}{F} = 4(0.05) = 0.2$

3.10.43.

(a) $L(x) = f(1) + f'(1)(x-1) = 5 - 1(x-1) = 6 - x$
$f(0.9) \approx L(0.9) = 5 - (0.9 - 1) = 5.1$
$f(1.1) \approx L(1.1) = 5 - (1.1 - 1) = 4.9$

(b) Notice that $(f')'(x) < 0$, hence $f''(x) < 0$, so f is concave down (Think for example $\sqrt{x}$, we'll discuss that more in section 4.3), which means that the linear approximations will be overestimates (in other words, the tangent line to f at 1 will be above the graph of f; again, think of the case $\sqrt{x}$)

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Chapter 3 – Review

TRUE–FALSE.

(1) **TRUE**
(2) **FALSE**
(3) **TRUE**
(4) **TRUE**
(5) **FALSE** ($\frac{f'(\sqrt{x})}{2\sqrt{x}}$)
(6) **FALSE** (e^2 is a constant, so 0)
(7) **FALSE** ($y' = \ln(10)10^x$, exponential rule)
(8) **FALSE** ($\ln 10$ is a constant, so 0)
(9) **FALSE** ($2\tan(x)\sec^2(x)$)
(10) **FALSE** ($(2x+1)\frac{x^2+x}{|x^2+x|}$; Write $\left|x^2+x\right| = \sqrt{(x^2+x)^2}$ and use the chain rule)
(11) **TRUE**
(12) **TRUE** (f is a polynomial of degree 30, so its 31st derivative is 0)
(13) **TRUE** (By the quotient rule and (11))
(14) **FALSE** ($y - 4 = -4(x+2)$; it's not even the equation of a line!)
(15) **TRUE** ($= g'(2) = 5(2)^4 = 80$)

3.R.68.

(a) $\sin(2x) = 2\sin(x)\cos(x)$
(b) $\cos(x+a) = \cos(x)\cos(a) - \sin(x)\sin(a)$ (The important thing here is that you differentiate with respect to x, leaving a constant)

3.R.87.

$$v(t) = s'(t) = -Ace^{-ct}\cos(\omega t+\delta) - A\omega e^{-ct}\sin(\omega t+\delta) = -Ae^{-ct}\left(c\cos(\omega t+\delta) + \omega\sin(\omega t+\delta)\right)$$

$$a(t) = v'(t) = Ace^{-ct}\left(c\cos(\omega t+\delta) + \omega\sin(\omega t+\delta)\right) - Ae^{-ct}\left(-c\omega\sin(\omega t+\delta) + \omega^2\cos(\omega t+\delta)\right)$$

3.R.94. $y(t) = 100 \times 2^{-\frac{t}{5.24}}$

(a) $y(20) = 100 \times 2^{\frac{-20}{5.24}} \approx 7.1$ mg
(b) $t = \frac{5.24\ln(100)}{\ln(2)} \approx 34.81$ years

3.R.99.

(1) $D^2 = x^2 + y^2$ (Typical Pythagorean theorem problem; Draw a right triangle in the shape of an L, and let x be the bottom side, y be the left-hand-side, and D be the hypothenus)
(2) $2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$
(3) $x = 3 \times 15 = 45$ (velocity $\times$ time), $y = 45 + 5 \times 3 = 60$ (initial height + velocity $\times$ time), $\frac{dx}{dt} = 15$, $\frac{dy}{dt} = 5$, and $D = \sqrt{x^2+y^2} = \sqrt{45^2+60^2} = \sqrt{5625} = 75$ which gives:

$$\frac{dD}{dt} = \frac{45 \times 15 + 60 \times 5}{75} = 13$$

3.R.101.

(1) $\tan(\theta) = \frac{400}{x}$ (Typical trigonometry-problem. Draw another triangle in the shape of an L, let 400 be the left-hand-side, x be the bottom, and the angle on the right be θ)

(2) $\sec^2(\theta)\frac{d\theta}{dt} = -\frac{400}{x^2}\frac{dx}{dt}$

(3) $x = 400\sqrt{3}$ (redraw the same triangle, but this time with $\theta = \frac{\pi}{6}$), $\frac{d\theta}{dt} = -0.25$, and $\theta = \frac{\pi}{6}$, which gives:

$$\frac{dx}{dt} = 400$$

3.R.105. The area of the window is given by $y = x^2 + \frac{\pi}{2}\left(\frac{x}{2}\right)^2 = \left(1 + \frac{\pi}{8}\right)x^2$. Then:

$$dy = \left(1 + \frac{\pi}{8}\right)2xdx = \left(1 + \frac{\pi}{8}\right)(120)(0.1) = 12 + \frac{3\pi}{2} \approx 16.71$$

3.R.111. $f'(2x) = \frac{x^2}{2}$, so $f'(x) = \frac{\left(\frac{x}{2}\right)^2}{2} = \frac{x^2}{8}$

Section 4.1: Maximum and Minimum Values

4.1.6.

- Absolute maximum: Does not exist (**NOT** 5)
- Absolute minimum: $f(4) = 1$
- Local minimum: $f(2) = 2$, $f(4) = 1$
- Local maximum: $f(3) = 4$, $f(6) = 3$

4.1.7, 4.1.10, 4.1.14. Ask me about that during office hours!

4.7.41. $f'(\theta) = -2\sin(\theta) + 2\sin(\theta)\cos(\theta)$, which gives $\theta = \pi m$ or $\theta = 2\pi m$, which can be just written as $\boxed{\theta = \pi m}$ (m is an integer)

4.7.43. $f'(x) = 2xe^{-3x} - 3x^2e^{-3x}$, which gives $\boxed{x = 0}$ and $\boxed{x = \frac{2}{3}}$

4.1.51. $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x^2 - x - 2) = 12x(x-2)(x+1)$, which gives $x = -1, 0, 2$. Candidates: $f(-2) = 33$, $f(3) = 28$ (endpoints), $f(0) = 1$, $f(-1) = -4$, $f(2) = -31$. Absolute maximum: $\boxed{f(-2) = 33}$, Absolute minimum: $\boxed{f(2) = -31}$

4.1.57. See document 'Solution to 4.1.57' on my website!